

Self-foldability of monohedral quadrilateral origami tessellations

Thomas C. Hull and Tomohiro Tachi

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Abstract

This paper concerns the self-foldability of rigidly-foldable origami tessellations that are monohedral, specifically crease patterns that are single quadrilateral tile repeated. We use the mathematical definition of self-foldability from [Tachi and Hull \(2016\)](#): Given a rigidly-foldable crease pattern with n creases, we define its *configuration space* $S \subset \mathbb{R}^n$ to be the set of points $x \in \mathbb{R}^n$ where the i th coordinate of x is the folding angle of the i th crease in a rigidly-folded state of the crease pattern. A *rigid folding* is defined to be a C^1 curve $\boldsymbol{\rho}(t)$ in S for $t \in [0, s]$ where $\boldsymbol{\rho}(0)$ is the initial state of the folded object and $\boldsymbol{\rho}(s)$ is the target state. For self-folding, we want to find rotational forces to place on the creases that will force the crease pattern to fold from the initial state to the target state along the curve $\boldsymbol{\rho}(t)$, and to do that we consider vector fields on our configuration space S . A vector field \mathbf{f} on S is called a *driving force* for the rigid folding $\boldsymbol{\rho}(t)$ if $\boldsymbol{\rho}'(t) \cdot \mathbf{f}(\boldsymbol{\rho}(t)) > 0$ for all $t \in [0, s]$.

Define $d(t) = \boldsymbol{\rho}'(t) \cdot \mathbf{f}(\boldsymbol{\rho}(t))$, which is called the *forward force* of \mathbf{f} along $\boldsymbol{\rho}(t)$. This quantity measures the angle between $\mathbf{f}(\boldsymbol{\rho}(t))$ and $\boldsymbol{\rho}'(t)$ and thus represents the amount of force $\mathbf{f}(\boldsymbol{\rho}(t))$ contributes to pushing in the direction of $\boldsymbol{\rho}'(t)$ along the rigid folding curve $\boldsymbol{\rho}(t)$.

Now, for a point $x \in S$ let $T_x S$ denote the tangent space of S at x . We say that a rigid folding $\boldsymbol{\rho}(t)$ is *self-foldable* by a driving force \mathbf{f} if the forward force $d(t)$ is a local maximum on $T_{\boldsymbol{\rho}(t)} S$ for $t \in [0, s]$. Maximizing $d(t)$ among the tangent vectors insures that \mathbf{f} will push the folding in the direction of the tangent $\boldsymbol{\rho}'(t)$ and thus along the curve $\boldsymbol{\rho}(t)$. If $\boldsymbol{\rho}(t)$ is the only rigid folding that is self-foldable by \mathbf{f} , then we say $\boldsymbol{\rho}(t)$ is *uniquely self-foldable* by \mathbf{f} .

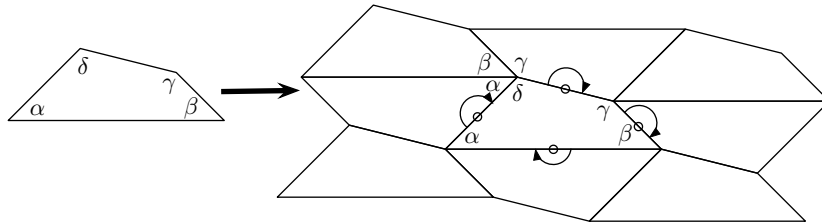


Figure 1: A group p2 non-flat-foldable origami tessellation.

For origami tessellations generated by a single quadrilateral tile, the classic Miura-ori is one example we will consider. A more general class of such tessellations, however, can be made

from any quadrilateral by repeatedly rotating about the midpoint of each side of the tile by 180° . Using wallpaper group notation, we will call such a quadrilateral tiling a *group p2* tiling; see Figure 1. If we also require that the opposite angles of the quadrilateral tile be supplementary, then Kawasaki's Theorem will be satisfied at each vertex of the resulting tessellation, implying that the tessellation will be (locally) flat-foldable; see Figure 2.

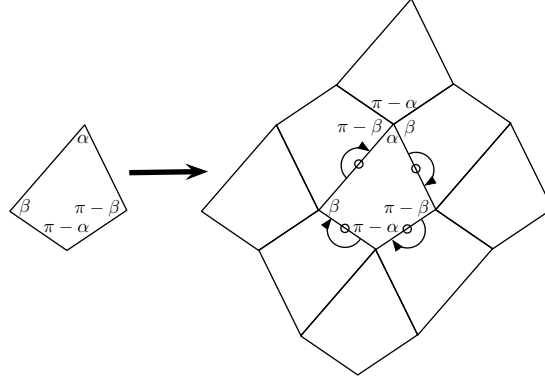


Figure 2: A group p2 flat-foldable origami tessellation.

The main results of this paper are the following:

Theorem 1 *Let C be a monohedral quadrilateral origami tessellation that is rigidly foldable with rigid folding $\mathbf{p}(t)$ for $t \in [0, s]$ for any initial folded state $\mathbf{p}(0)$ and target state $\mathbf{p}(s)$.*

- (a) *If C is the Miura-ori, then $\mathbf{p}(t)$ is not uniquely self-foldable.*
- (b) *If C is a group p2 tiling and not locally flat-foldable with $\alpha \leq \beta < 90^\circ$ (as in Figure 1), then there exists a driving force \mathbf{f} that makes $\mathbf{p}(t)$ uniquely self-foldable.*
- (c) *If C is a group p2 tiling and locally flat-foldable with $\alpha < \beta < 90^\circ$ (as in Figure 2), then there exists a driving force \mathbf{f} that makes $\mathbf{p}(t)$ uniquely self-foldable.*
- (d) *If C is a group p2 tiling and locally flat-foldable with $\alpha = \beta < 90^\circ$, then $\mathbf{p}(t)$ is not uniquely self-foldable.*

A classic example of case (d) in Theorem 1 is the Yoshimura crease pattern, where each tile is an isosceles trapezoid.

References

Tomohiro Tachi and Thomas C. Hull. Self-foldability of rigid origami. *ASME Journal of Mechanisms and Robotics*, 9(2):021008–021017, 2016.